Decision Aids for Asset-to-Objective Allocation

R. Balart, J.S. Byrnes[†] D. Cochran[‡] M. Larkin[§] and W. Moran[¶]
Prometheus, Inc.
21 Arnold Avenue
Newport, RI 02840

Abstract

Mathematical mechanisms have recently been proposed to address large scale optimization problems in which optimal or nearly optimal allocations of a hierarchically organized collection of assets to a hierarchically organized set of objectives are to be identified. This paper describes an automated decision aid that uses the Extended Dependency Model (EDM) for evaluation of costs and benefits within hierarchical structures and a Genetic Algorithm (GA) for optimization. Design of a graphical user interface to support user formulation of asset allocation problems involving complex hierarchical structures of assets and objectives is described, and examples involving tasking of aircraft assets in tactical scenarios are presented.

1 Introduction

Many real-world decision problems involve allocation of elementary assets among elementary objectives in order to achieve a high-level goal. Financial investment provides one example. Elementary assets (e.g., dollars) are to be allocated among several investment options in an overall investment strategy. Another example is determination of which players on a baseball team should start against a particular opposing team under given playing conditions. Assignment of military aircraft to targets in a manner that best supports an overall campaign goal is the allocation problem that motivated much of the work presented in this paper, and an example drawn from this situation is described in detail below.

Commonly, elementary objectives contribute to high-level goals in a manner that can be captured by a hierarchical dependency model. A mathematical tool called the extended dependency model (EDM) provides a mechanism for computing the importance of low-level objectives to achieving higher-level objectives [2]. It can also be used to compute the importance of elementary assets within a hierarchically organized collection of assets.

This paper describes an approach to asset-to-objective allocation problems based on optimization of a payoff function in which high-level importances of elementary assets and objectives are computed with EDM. A prototype decision aid tool in which this approach is realized is also described. Since typical payoff functions incorporate a large amount of data about costs and efficiencies of elementary assets (or sets of elementary assets) in achieving elementary objectives, and because EDM requires a potentially complicated parametric description of the hierarchy on which it operates, design of the tool's user interface has proven to be particularly important. Also, the combinatorics of the problem of finding the optimal allocation strategy become intractable by exhaustive search for even small problems. A genetic algorithm approach for finding nearly optimal solutions has been adopted for the decision aid. Both of these practical aspects of the problem are discussed in the paper.

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[†] J.S. Byrnes is also affiliated with the Department of Mathematics; University of Massachusetts; Boston, MA 02125.

[‡] D. Cochran is also affiliated with the Department of Electrical Engineering; Arizona State University; Tempe, AZ 85287-7206.

[§] M. Larkin is also affiliated with the Department of Applied Mathematics; Brown University; Providence, RI 02912.

[¶] W. Moran is also affiliated with the Discipline of Mathematics; Flinders University of South Australia; Adelaide, 5001 South Australia.

2 Mathematical Formulation

The asset-to-objective allocation problem begins with a collection of elementary assets $\{B_1, ..., B_m\}$ and a set of elementary objectives $\{T_1, ..., T_n\}$. Each asset B_i has an importance R_i associated with it, and each objective T_j has an importance ρ_j associated with it. These importances will generally be evaluated indi-

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rectly in terms of dependency models on hierarchical structures of assets and objectives, as will be discussed further below. The goal is to determine optimal or nearly optimal *allocation strategies* in which each B_i is assigned to some T_j (or possibly left unassigned), where optimality is with respect to a payoff function which incorporates the importances of the assets and objectives as well as additional information about costs and effectivenesses associated with the strategy. Examples of such information, which may include both "hard" data and figures derived from "expert opinion," are:

- The *efficiency* $p_{i,j}$ of asset B_i in achieving objective T_j.
- The *cost* $c_{i,j}$ of allocating B_i to T_j.
- The *joint efficiency* $\phi_{i_1,i_2,j}$ added by simultaneously assigning assets T_{i_1} and T_{i_2} to objective T_j.

2.1 Hierarchical dependency models

As mentioned above, it is frequently possible to quantify the overall importance of an elementary asset or objective by considering its role in a hierarchical structure. A hierarchy of objectives is depicted in figure 1. Elementary objectives appear at the lowest level (the third level in the example pictured) of the hierarchy where they are grouped according to which objectives at the next higher level of the hierarchy they affect. In the diagram, elementary objectives $\{T_{3,1,1}, ..., T_{3,1,n(3,1)}\}$ contribute to objective $T_{2,1,1}$ at the second level of the hierarchy. Each of the elementary objectives $T_{3,1,j}$ is assigned a *relative importance* $w_{3,1,j} \geq 0$ with the condition that

$$\sum_{j=1}^{n} w_{3,1,j} = 1$$

Proceeding up the hierarchy, relative importances are assigned to the objectives at each level to describe how they contribute to the objectives at the next higher level. Such an *extended dependency model* (EDM) allows the importance of each elementary objective T_j to be quantified in terms of the overall (i.e., highest level) objective as the product ρ_j of the weights above T_j in the hierarchy. The EDM may be applied to a hierarchy of assets in a similar way. The role of the EDM in a military aircraft allocation problem is illustrated later in this paper.

2.2 Representation of strategy

An allocation strategy can be represented as a binary $n \times m$ matrix X whose $(i,j)^{\text{th}}$ entry is one if B_i is allocated to T_j and is zero otherwise. In many situations, an asset can be assigned to at most one objective. This constraint can be introduced by requiring the row sums of X to be less than or equal to one:

$$\sum_{j=1}^{m} x_{i,j} \leq 1 \tag{1}$$

2.3 The payoff function

The nature of the payoff function to be optimized obviously depends heavily on the particular application. For the problem of determining optimal or nearly optimal allocation strategies, the payoff function must depend on the strategy, and will thus be denoted by $J(X)$.

If the efficiency $p_{i,j}$ is interpreted as the probability of the event "elementary asset i will achieve elementary objective j if so allocated" and these events are statistically independent, then the probability of elementary objective j being achieved by allocation strategy X is

$$\Pi_j = 1 - \prod_{i=1}^{m}(1 - x_{i,j}p_{i,j}) \tag{2}$$

Thus the payoff function generally contains a term of the form

$$U(X) = \sum_{i=1}^{n} \rho_j \Pi_j \tag{3}$$

which weights the probability each elementary objective is achieved by its importance. The degree to which the events fail to be independent may be captured by other terms which quantify the additional benefit of allocating two or more assets to the same objective, for example.

Another term in $J(X)$ typically measures the cost of the strategy:

$$Y(X) = \sum_i \sum_j c_{i,j} x_{i,j} \tag{4}$$

3 Optimization

Assuming that each asset can be allocated to at most one objective, a collection of n assets and m

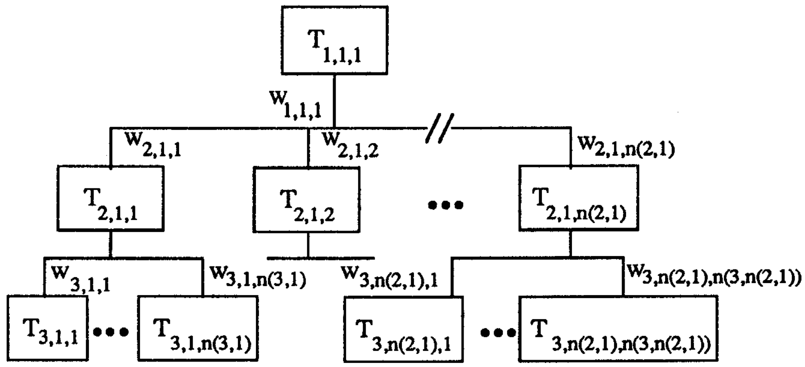

Figure 1: Structure of a hierarchical dependency model. This figure is an objective hierarchy; asset hierarchies are similar.

objectives yields $(m+1)^n$ possible allocation strategies. Exhaustive search for an optimal strategy becomes inviable for even modest collections of assets and objectives and computationally simple cost functions. A problem involving 15 assets and 10 objectives in which the cost function requires 1 millisecond to compute would require over 130 years to optimize by exhaustive search, for example.

Several efficient search methods are known for addressing combinatorially large optimization problems. These include gradient methods, simulated annealing, and genetic algorithms. Among these possibilities, genetic algorithms offer some features that are particularly attractive for problems involving complicated non-convex payoff functions. They are faster than simulated annealing and less likely to stagnate in local minima than gradient methods. In addition, under assumption (1), allocation strategies are conveniently represented as "genes" in a way that will be described below. The principles of GA's, which seek the "fittest" genes (i.e., best strategies in terms of J) by a Darwinian paradigm, are presented in [3].

4 A Tactical Decision Aid

The approach to asset allocation described above has been instantiated in a prototype interactive decision aid for allocation of tactical air assets. In the context of this application, the payoff function $J(X)$ is called the "measure of effectiveness" (MOE) of the allocation strategy X. The decision aid allows a user to evaluate the MOE of a proposed strategy and will also automatically provide the user with a list of candidate strategies having optimal or nearly optimal MOE's.

The payoff function for this application and the decision aid's performance in an example allocation scenario are discussed below.

4.1 The MOE

The form of the payoff function used is

$$J(X) = AU(X) + BV(X) - \Gamma Y(X)$$

where $U(X)$ is the benefit term described by equations (2) and (3) and $Y(X)$ is the cost/risk term given in equation (4). The third term incorporates data $\xi_{i_1,i_2,j}$ describing the marginal benefit of allocating both B_{i_1} and B_{i_2} to T_j:

$$V(X) = \sum_{i_1=1}^{m} \sum_{i_2=1}^{m} \sum_{j=1}^{n} \xi_{i_1,i_2,j} x_{i_1,j} x_{i_2,j}$$

The $\xi's$ can be negative (it is undesirable to pair a stealth bomber with a more detectable aircraft for a surprize attack, for example). The weights A, B, and Γ allow the importance of each term to be adjusted in the overall MOE.

4.2 Air asset allocation example

A simple example illustrating the function of the decision aid follows. A set of seven elementary aircraft assets is available:

- B_1 = B2 stealth bombers
- B_2 = B52 heavy bombers

- B_3 = F111 fighter-bombers
- B_4 = F16 multi-mission fighters
- B_5 = F16 multi-mission fighters
- B_6 = F4-G wild weasels
- B_7 = F4-G wild weasels

These assets are to be employed in a conflict where the overall objective "Degrade the enemy's war sustaining capabilities" is supported by numerous sub-objectives in the objective hierarchy partially depicted in figure 2. One cluster of elementary objectives is:

- T_1 = Port, AAA protection, relative importance 0.20
- T_2 = Railyard, no protection, relative importance 0.10
- T_3 = Large petroleum reserve, SAM protection, relative importance 0.35
- T_4 = Warehouses, AAA protection, relative importance 0.10
- T_5 = Small petroleum reserve, SAM protection, relative importance 0.20
- T_6 = Bridges, SAM protection, relative importance 0.05

Using a substantial database describing the individual and pairwise effectivenesses and costs of each elementary asset against each elementary objective, a GA optimizer produced several candidate solution genes. A gene is a vector of length m (number of assets) in which the i^{th} element is the index j if B_i is to be allocated to T_j and zero if the asset is not used. Under assumption (1), every strategy X can be represented in this way.

The strategies with the highest MOE's were (1,2,4,5,3,3,3) and (3,2,1,5,4,0,5), which are depicted in figure 3. Readers familiar with the nature of these assets and objectives can judge the reasonableness of the solutions obtained (bearing in mind that they simply reflect the underlying data used). It is interesting to note that the second-best strategy obtained does not use one of the assets.

4.3 Role of a GUI

It is noteworthy that a substantial amount of data (e.g., objective and asset importances, effectivenesses of assets against objectives, costs, etc.), some of which may have to be distilled from qualitative "expert information," and a substantial parametric structure (e.g., asset and objective hierarchies) underlie problems involving allocation of air assets. A well designed graphical user interface (GUI) can significantly enhance a typical user's ability to develop a suitable parametric description of the problem to be addressed by the decision aid. An GUI based on X-Windows for use in this application was described in [1].

5 Concluding Remarks

The goal of this paper has been to suggest how a decision aid for asset allocation problems might serve to evaluate candidate strategies and, in conjunction with a powerful optimization program, even propose candidate strategies having high payoffs. It has also pointed out how the EDM can be used to quantify importances of objectives and assets and that a useful decision aid depends on the ability of the user to formulate the problem to be solved in terms that can provide a parametric description of the problem to be solved; the user interface design is thus crucial to the utility of the decision aid.

Ongoing work is exploring the ability of the formulation in this paper to be used in real-time allocation applications in which allocation strategies must be determined in minutes, seconds, or even milliseconds. Suitable parallelization of GA's is one thrust in this work.

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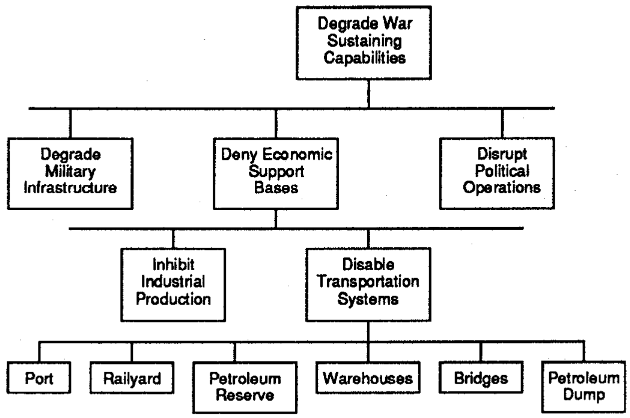

Figure 2: A partial objective hierarchy in which the overall objective depends on the six elementary objectives listed in the text.

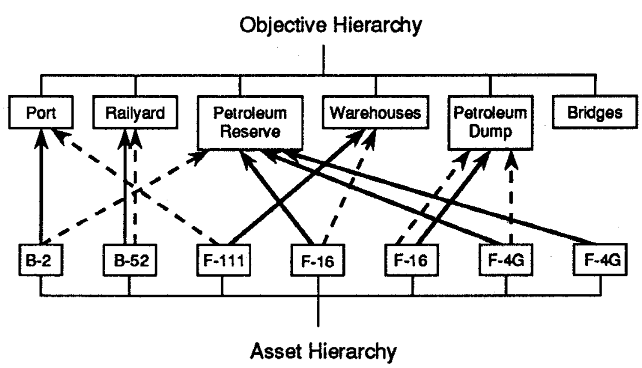

Figure 3: Two near-optimal asset-to-objective allocation strategies identified for a tactical air example. The solid arrows represent gene (1 2 4 5 3 3 3) with MOE 0.584; the dashed arrows represent gene (3 2 1 5 4 0 5) with MOE 0.577. Note that the second solution leaves one asset unallocated.